Asymmetric Sequential Search*

Carmen Astorne-Figari † Aleksandr Yankelevich ‡

This version: September 16, 2013

Abstract

This note explores asymmetries in the way consumers sample prices in a simple variation of Stahl's (1989) seminal model of sequential search. In the note, we characterize a unique equilibrium in which a firm that caters to more local consumers selects prices from a distribution which first order stochastically dominates that of its rival and contains mass at the upper bound of firm price distributions. Both firms exhibit higher prices as the proportion of consumers local to one firm rises, though surprisingly, at the limit, the Diamond paradox may not manifest.

Keywords: Sequential Consumer Search; Oligopoly; Price Dispersion

JEL Classification Numbers: D43, D83, L13

---

*The analysis and conclusions set forth are those of the authors and do not necessarily represent the views of the Federal Communications Commission, other Commission staff members, or the U.S Government.

†Department of Economics, University of Memphis, 427 Fogelman Admin Building, Memphis, Tennessee 38152; e-mail address: cmstrnfg@memphis.edu

‡Corresponding author. Federal Communications Commission, 445 12th Street, SW, Washington, DC 20554. e-mail address: aleks.yankelevich@fcc.gov. phone: +1 (917) 346-5269

1 Introduction

Recent work in sequential search tells us that search order matters, but that its effect on pricing depends fundamentally on the underlying modeling framework. For instance, using the heterogeneous product match setup developed by Wolinsky (1986), Armstrong et al. (2009) show that when all consumers begin search at a prominent firm, that firm will charge a lower price than its less prominent rivals.[1] Conversely, Arbatskaya (2007), working in a model with homogeneous products, but with heterogeneous costs of search, finds that when search is ordered, prices must decline in the order in which they are sampled.[2]

These seemingly contrary results suggest that search models should be tailored to explain the appropriate economic phenomenon. Arbatskaya's (2007) model applies when price is the only deciding factor in a product purchase, as might be the case in a farmer's market, whereas the framework of Armstrong et al. (2009) is better suited to a setting where consumers care about other product qualities, such as when heterogeneous brands vie for shelf space. Moreover, neither of these articles attempts to address non-random search where different groups of consumers may follow different search orders. Such a situation has broad application in the context of asymmetric location oligopoly. Consider as an example, two competing nationwide, homogeneous product retailers whose main difference is in the number of store locations (such a situation might arise in an industry where one firm had a first-mover advantage). If some consumers find it costly to visit a non-local store, the retailer with more nationwide stores is more likely to be sampled first by more, though not necessarily *all*, consumers. Another application concerns spatially differentiated retail establish-

[1] Zhou (2010) obtains similar results by extending this setup to completely ordered search.
[2] Arbatskaya's search cost setup is related to the models of Benabou (1993) and Stahl (1996).

ments selling essentially the same basket of goods—e.g., convenience store chains sell many of the same goods as larger discount retailers or supermarkets, but cater primarily to individuals with a high opportunity cost of shopping elsewhere.

We cast our study in the sequential search framework of Stahl (1989), in which the good is homogeneous while consumer search cost heterogeneity is embodied via two consumer types: shoppers who have no opportunity cost of time and non-shoppers who engage in costly sequential search.[3] We adjust the frequently used assumption that all consumers freely and randomly choose the first firm to sample by supposing that non-shoppers always sample their local firm first. Our main finding is that there is a unique equilibrium in which the price distribution of the firm associated with a larger local population first order stochastically dominates that of its competitor. This finding generalizes a well known result established by Narasimhan (1988),[4] but differs from it in two key respects: (i) because we do not exogenously impose a captive segment of consumers, the upper bound of price distributions can fall well below the price that would prevail in a monopoly, and (ii) the limiting result when there are no "informed" consumers is not necessarily that of Diamond (1971).

2 The model

Two firms, labeled 1 and 2, sell a homogeneous good. Firms have no capacity constraints and an identical constant cost of zero of producing one unit of the good. There is a unit mass of almost identical consumers with inelastic (unit) demand and

[3]In order to calculate explicit solutions and fully characterize our equilibria, we borrow from Janssen et al. (2005) by assuming that consumers hold inelastic demands.

[4]This result is explored further in Deneckere et al. (1992) and Jing and Wen (2008). However, unlike this note, both articles follow Narasimhan by supposing that non-shoppers behave like the uninformed consumers in Varian's (1980) model of sales—that is, their reservation price is not derived optimally from the equilibrium price distribution.

valuation $v > 0$ for the good. A proportion $\mu \in (0, 1)$ of consumers are shoppers who have no cost of search. The remaining $1 - \mu$ non-shoppers, pay a positive search cost $c \in (0, v)$ for each firm they visit except for their local firm, which they search first. A fraction $\lambda \in [0, 1]$ of the $1 - \mu$ non-shoppers is local to firm 1, while the remaining $1 - \lambda$ are local to firm 2. Non-shoppers search sequentially with costless recall.

Firms and consumers play the following game. First, firms 1 and 2 simultaneously choose prices taking into consideration their beliefs about the rival firm's pricing strategy and about consumer search behavior. A pricing strategy consists of a price distribution F_i over a support with lower (upper) bounds $\underline{p}_i$ ($\bar{p}_i$), where $F_i(p)$ is the probability that firm $i = 1, 2$ offers a price no higher than p. Once prices have been realized, consumers choose optimal search strategies given their beliefs about each F_i. Parameters v, c, μ, λ, as well as the rationality of all agents in the model are commonly known.

3 Equilibrium analysis

We analyze the Sequential Equilibrium of this game. In this context, Sequential Equilibrium requires that non-shoppers who observe an off-equilibrium price at their local firm treat such deviations as "mistakes" when forming beliefs about the non-local firm's strategy. Thus, non-shoppers believe that the non-local firm plays its equilibrium strategy at all information sets.

3.1 Consumer behavior

Shoppers, who can search freely, will sample both firms before making their purchase decision. Moreover, because we impose a certain search order on non-shoppers, it suffices to consider their decision regarding whether or not to search the non-local firm. It is well established that in the framework described above, the optimal search rule is for a non-shopper who has freely observed the price at firm j to continue search if and only if the observed price is higher than a reservation price, r_i, which makes him indifferent between searching firm i and stopping. This reservation price is then defined as the solution to

$$\int_{\underline{p}_i}^{r_i} (r_i - p)\mathrm{d}F_i(p) = \int_{\underline{p}_i}^{r_i} F_i(p)\mathrm{d}p = c \tag{1}$$

Note that reservation price r_i corresponds to consumers who begin their search at firm j and vice versa because consumers who begin at firm j must decide whether or not to search firm i based on the price they observed at firm j and their beliefs about firm i's pricing strategy.

3.2 Firm pricing

Before characterizing the equilibrium of the game, we need to place certain limitations on the way that firms may price in equilibrium.

Proposition 1. *In equilibrium, the supports of the firm pricing distributions are the same and do not have any breaks. Both supports are bounded from above by* $\bar{p} = \min\{v, r_1, r_2\}$ *and at most one firm may have one atom at* $\bar{p}$*. Suppose that an atom exists in equilibrium. Then non-shoppers who sample firm i first, must stop searching after observing a price of* r_j *unless* $v < r_j$*.*

The proof of this proposition, is contained in the Appendix. The proposition tells us that even if one firm is local for a larger proportion of non-shoppers, it would nonetheless never offer a price higher than the largest possible price of its competitor, nor a price high enough to induce its local non-shoppers to search further. Note that because a price equal to r_j and strictly higher than v is only observed off the equilibrium path, in equilibrium, non-shoppers whose first observation equals r_j stop.

We next state our main result by describing the unique Sequential Equilibrium of this game. The existence of an equilibrium follows by construction below, whereas uniqueness proceeds directly from Proposition 1. Consider first $\lambda = 1/2$. This is equivalent to random non-shopper search. The resulting equilibrium has been fully established in the literature (e.g., Janssen et al. 2005, Proposition 1) and we do not reproduce it here. Going forward, without loss of generality, we suppose that $\lambda \in (1/2, 1]$—that is, firm 1 is local for a larger proportion of non-shoppers.

Proposition 2. *Suppose that $\lambda \in (1/2, 1]$. There exists a unique Sequential Equilibrium where both firms distribute prices over support $\left[\underline{p}, \bar{p}\right]$, where $\bar{p} = \min\{v, r_2^*\}$, $\underline{p} = \frac{\lambda(1-\mu)}{\lambda(1-\mu)+\mu}\bar{p}$ and r_1^* and r_2^* are the equilibrium reservation prices for non-shoppers local to firm 2 and firm 1 respectively. $r_2^* = r_2(\mu, \lambda, c) \equiv c\left\{1 - \frac{\lambda(1-\mu)}{\mu} \ln\left[1 + \frac{\mu}{\lambda(1-\mu)}\right]\right\}^{-1}$ if $r_2(\mu, \lambda, c) \leq v$ and ∞ otherwise, while $r_1^* = \infty$. Firm 1 distributes prices according to $F_1(p) = \frac{1-\lambda(1-\mu)}{\mu}\left(1 - \frac{\underline{p}}{p}\right)$ on $\left[\underline{p}, \bar{p}\right)$ with $\Pr(p_1 = \bar{p}) = \frac{(1-\mu)(2\lambda-1)}{\lambda(1-\mu)+\mu}$ while firm 2 distributes prices according to $F_2(p) = \frac{\lambda(1-\mu)+\mu}{\mu}\left(1 - \frac{\underline{p}}{p}\right)$.*

Proof. In equilibrium, a firm must be indifferent between any price in its support. Therefore, for any p_i in the support of F_j, solving $\mathrm{E}\,\pi_i\left(\underline{p}\right) = \mathrm{E}\,\pi_i\left(p_i, F_j\left(p_i\right)\right)$ for F_j, $i \neq j \in \{1, 2\}$, yields firm distribution functions:

$$F_1(p) = \frac{1 - \lambda(1-\mu)}{\mu}\left(1 - \frac{\underline{p}}{p}\right) \leq \frac{\lambda(1-\mu) + \mu}{\mu}\left(1 - \frac{\underline{p}}{p}\right) = F_2(p) \qquad (2)$$

where the inequality follows from the assumption $\lambda > 1/2$ and is strict for $p > \underline{p}$. Moreover, because $\bar{p}_1 = \bar{p}_2 = \bar{p}$, this implies that F_1 has an atom at $\bar{p}$.

Setting $F_2(\bar{p}) = 1$ to solve for $\underline{p}$ in terms of $\bar{p}$ and substituting into $F_2(p)$ gives

$$F_2(p) = \frac{\lambda(1-\mu)+\mu}{\mu}\left[1 - \frac{\lambda(1-\mu)}{\lambda(1-\mu)+\mu}\frac{\bar{p}}{p}\right]. \tag{3}$$

When $r_2 \leq v$, $\bar{p} = r_2$. Optimal search requires that Equation 1 holds. Substituting Equation 3 into Equation 1 and integrating to solve for r_2, we get

$$r_2(\mu, \lambda, c) = c\left\{1 - \frac{\lambda(1-\mu)}{\mu}\ln\left[1 + \frac{\mu}{\lambda(1-\mu)}\right]\right\}^{-1} \tag{4}$$

If $r_2(\mu, \lambda, c) \leq v$, r_2^* is defined by Equation 4. Because firms are not concerned with prices above v, if $r_2(\mu, \lambda, c) > v$, we define r_2^* to be positive infinity. Likewise, the inequality in Equation 2 allows us to define r_1^* to be positive infinity as well. □

In equilibrium, non-shoppers search their local firm and make a purchase there, whereas shoppers purchase from the firm with the lower price. Because non-shoppers always buy from their local firm, it is more costly for firm 1 to lower its price than it is for firm 2. Firm 1 takes advantage of its location by running fewer sales and pricing higher on average. When the cost of non-shopper search beyond the local firm is infinite, as is implicit in the baseline model of Narasimhan (1988), $\bar{p} = v$. However, as long as c is sufficiently low to make $r_2^* < v$, the monopoly price could never prevail when shoppers exist in the market.

We would like to know how our model behaves as the location asymmetry approaches that of a completely ordered search framework such as that of Arbatskaya (2007). First, note that $r_2(\mu, \lambda, c)$ is increasing in $\lambda \in (1/2, 1]$.[5] Knowing that $\bar{p}$ is non-decreasing in λ, for any $p \in [\underline{p}, \bar{p})$, it is easy to show that $F_1(\mu, \lambda; p)$ and $F_2(\mu, \lambda; p)$ are decreasing in λ.[6] That is, for $\lambda \in (1/2, 1]$, the price distributions for

[5]This follows directly from the fact that for all $A \in (0, \infty)$, the function $A\ln(1+1/A)$, is strictly increasing in A. Moreover, $\lim_{A\to 0} A\ln(1+1/A) = 0$ and $\lim_{A\to\infty} A\ln(1+1/A) = 1$. Setting $A = \lambda(1-\mu)/\mu$ we see that $\partial r_2(\mu, \lambda, c)/\partial\lambda > 0$ for $\lambda \in (1/2, 1]$.

[6]For the sake of completeness, we note that $r_2(\mu, \lambda, c)$ is decreasing in μ, while, $F_1(\mu, \lambda; p)$ and $F_2(\mu, \lambda; p)$ are non-decreasing in μ.

both firms at a higher λ first order stochastically dominate their lower λ counterparts. Clearly, as firm 1 gains local non-shoppers, its profit increases. However, even as firm 2 loses local non-shoppers, its ability to charge shoppers higher prices implies that its profit loss is more than offset by firm 1's profit gain (firm 2's profit may even rise for sufficiently large μ). In the first application mentioned in the introduction, this finding suggests that nationwide firms competing in multiple, separable local markets with varying market shares are better off maintaining asymmetric shares across regions than they would be by sacrificing high share in one region in exchange for market gains in a low share region.

When $\lambda = 1$, all non-shoppers search firm 1 first. Nevertheless, Propositions 1 and 2 apply, and as might be expected, price dispersion persists because $\mu > 0$. That is, as long as firms can capture a mass of consumers by undercutting their rivals' prices, below monopoly prices will be observed with positive probability. Surprisingly, as the next proposition will show, when search is completely ordered (that is, $\lambda = 1$), monopoly pricing may not prevail even if $\mu = 0$.

Proposition 3. *Suppose $\lambda = 1$ while $\mu = 0$. The set of pure strategy equilibria*[7] *can be characterized as follows: $p_1 \in [0, v]$. If $p_1 \in [0, v)$, $p_2 = p_1 - c$, $r_2^* = p_1 = p_2 + c < v$. If $p_1 = v$, $p_2 \in [v - c, \infty)$, and $r_2^* = \min\{v, \infty\}$.*

Proof. We restrict attention to pure strategy equilibria. Consider the strategies $p_1 \in [0, v)$ and $p_2 = p_1 - c$. Given consumer (correct) beliefs, $r_2(\mu, \lambda, c) = p_1 = p_2 + c < v$, so $r_2^* = r_2(\mu, \lambda, c)$. As in Proposition 1, existence entails that non-shoppers stop after observing the reservation price (otherwise firm 1 always has an incentive to lower its price). Consider any $p_2 > p_1 - c$. Then $p_1 < r_2(\mu, \lambda, c) = p_2 + c$ and firm 1 can

[7]In this case there is no completely mixed strategy equilibrium. Firm 1 will always play a pure strategy in equilibrium. Even though firm 2 can play a mixed strategy in equilibrium, this does not matter to consumers because they never observe its price.

profitably deviate to $p_2 + c$. Now suppose that $p_2 < p_1 - c$. But then $p_1 > r_2(\mu, \lambda, c)$ so all consumers leave firm 1 and never return. Thus, firm 1 can profitably deviate to $p_2 + c$. Finally, when $p_1 = v$, firm 2 may charge any price $p_2 > p_1 - c$. Given consumer (correct) beliefs, r_2^* can be appropriately defined as $\min\{v, \infty\}$, so firm 1 has no incentive to deviate. $\square$

When $\lambda = 1$, there are multiple pure strategy equilibria where firm 2 undercuts firm 1 by c. In such equilibria, because non-shoppers do not search, firm 1 would like to charge v. However, because firm 2 makes no profit, it can charge any price. By pricing below $v - c$, firm 2 raises the marginal benefit of search, causing r_2^* to fall below v. In order to keep its local non-shoppers from searching firm 2, firm 1 has to lower its price below v. Thus, an inactive firm can lead to below monopoly pricing when all consumers have to pay to sample prices beyond the first one.[8]

4 Conclusion

We have followed the spirit of the ordered search literature by supposing that non-shoppers sample their local firm first. We have not provided non-shoppers with an option to choose whether or not to search their local firm first at a discounted, albeit possibly positive search cost. Clearly, the present outcome would be a violation of Weitzman's (1979) Pandora's rule for the majority of non-shoppers if the first price sample were endogenously determined under the alternative framework. As such, one direction for future research is the characterization of an asymmetric, homogeneous product search equilibrium consistent with Pandora's rule.

[8] This result resembles the rational-expectations model of Arbatskaya (2007), where an inactive firm is required to sustain price dispersion in equilibrium.

Acknowledgments

The authors have benefited from insightful comments from Dmitri Kuksov, Stephanie Lau, David Levine, John Nachbar, Chakravarthi Narasimhan, Robert Pollak, and Paroma Sanyal. We also thank Ugur Ozdemir as well as seminar participants at the Southern Economic Association and Washington University in St. Louis.

Appendix

Proof of Proposition 1. We prove this proposition using a series of claims. The claims follow similarly to Propositions 2 through 5 in Narasimhan (1988). However, various complications arise because our consumer counterpart to Narasimhan's loyal segment follows an optimal search rule.

Claim 1. $v \geq \max\{\bar{p}_1, \bar{p}_2\} \geq \underline{p}_1 = \underline{p}_2 = \underline{p} \geq 0$.

Proof. Let γ be the proportion of non-shoppers who do not search after observing a price of r_j at their local firm i. Suppose $\underline{p}_1 < \underline{p}_2 \leq v$. Firm 1's profit on $[\underline{p}_1, \underline{p}_2)$ equals

$$p_1\{\mu + \lambda(1-\mu) + (1-\lambda)(1-\mu)[1 - F_2(r_1) + (1-\gamma)\Pr(p_2 = r_1)]\} \tag{5}$$

which is increasing in p_1, a contradiction. Now suppose that $\underline{p}_1 \leq v < \underline{p}_2$. Then for $p_1 \in [\underline{p}_1, v)$, firm 1's profit is given by Equation (5), which is increasing in p_1, so it must be the case that $\underline{p}_1 = v$. If $\underline{p}_1 = v < \underline{p}_2$, then firm 2 makes zero profit on its support, and would increase profits by shifting mass to v. If $v < \underline{p}_1 \leq \underline{p}_2$, then both firms make zero profits and either can increase profit by shifting mass to v, so $\underline{p}_2 \leq \underline{p}_1$. By a similar argument, $\underline{p}_2 \geq \underline{p}_1$ and $v \geq \underline{p}_1 = \underline{p}_2 = \underline{p}$.

Firms will not charge prices below zero because these yield negative profit.[9] Similarly, at prices above v firms make zero profit. As a result, all consumers in the market make a purchase. □

Definition 1. *We say that firms have a mutual atom when each firm has an atom at the same price. We say that firms have a mutual break when each firm's equilibrium support has a break over the same price interval.*

Claim 2. *There are no mutual atoms.*

Proof. Let α^S be the proportion of shoppers who buy from firm 1 after having observed the same price in both firms. Let α^N be the proportion of non-shoppers who buy from their local firm after having observed the same price in both firms. Suppose that both firms have a mutual atom at p. When $p_1 = p_2 = p$, firm 1's profit is given by

$$\begin{aligned} p\{\alpha^S\mu + \lambda(1-\mu)[\mathbb{I}_{p<r_2} + [\gamma + \alpha^N(1-\gamma)]\mathbb{I}_{p=r_2} + \alpha^N\mathbb{I}_{p>r_2}] \\ + (1-\lambda)(1-\mu)(1-\alpha^N)[(1-\gamma)\mathbb{I}_{p=r_1} + \mathbb{I}_{p>r_1}]\} \end{aligned} \tag{6}$$

where $\mathbb{I}$ is an indicator function. Suppose that firm 1 sets $p_1 = p - \varepsilon$ instead of p. Then profits become

$$(p-\varepsilon)\{\mu + \lambda(1-\mu) + (1-\lambda)(1-\mu)[(1-\gamma)\mathbb{I}_{p=r_1} + \mathbb{I}_{p>r_1}]\} \tag{7}$$

Expression (7) is larger than Expression (6) for ε sufficiently small.

Suppose firm 2 chooses a price other than p. Lowering the price charged never reduces the number of sales, so the loss to firm 1 from lowering the price by ε is at most ε. However, when p is charged with positive probability, lowering the price by ε will with positive probability lead to a gain and with complementary probability, at worst lead to a loss of ε. Therefore, by shifting its atom at p to $p - \varepsilon$ for sufficiently

[9] If $\underline{p} = 0$, then there must be zero density at $\underline{p} = 0$.

small ε, firm 1 increases its expected profit, a contradiction. By a similar argument, firm 2 will want to undercut a mutual atom for $\lambda \neq 1$.

For the case $1 - \alpha^S = \alpha^N = \lambda = 0$, firm 1 does not have a profitable deviation, but firm 2 does. □

Claim 3. *The only possible breaks in the equilibrium supports are:*

(i) if $r_j = \bar{p}_i < \bar{p}_j = \min\{r_i, v\}$ *there is a break at* $(\bar{p}_i, \bar{p}_j)$

(ii) if $r = r_i = r_j < \bar{p}_i = \bar{p}_j$ *there may be a mutual break with lower bound* r*, and*

(iii) if $r_i \neq r_j$ *and firm* i *has an atom at* r_j*, there may be a mutual break with lower bound* r_j*.*

Proof. Let S_1 and S_2 denote the equilibrium supports for firms 1 and 2 respectively. Let $\hat{p} = \inf(S_1 \cap S_2)$ and $\hat{\hat{p}} = \sup(S_1 \cap S_2)$. Define $H = (p^d, p^u) \in \text{int}(S_1 \cap S_2)$.

Suppose first, without loss of generality, that in equilibrium, firm 2 has no support over H, but that firm 1 does. Firm 1's expected profit at $p_1 \in H$ is

$$\begin{aligned} p_1\{\ & \mu[1 - F_2(p_1)] \\ & + \lambda(1-\mu)\{\mathbb{I}_{p_1<r_2} + [\gamma + (1-\gamma)[1 - F_2(p_1)]]\mathbb{I}_{p_1=r_2} + [1 - F_2(p_1)]\mathbb{I}_{p_1>r_2}\} \\ & + (1-\lambda)(1-\mu)\{[1 - F_2(r_1) + (1-\gamma)\Pr(p_2 = r_1)]\mathbb{I}_{p_1<r_1} \\ & + [1 - F_2(r_1)]\mathbb{I}_{p_1=r_1} + [1 - F_2(p_1)]\mathbb{I}_{p_1>r_1}\}\ \} \end{aligned} \tag{8}$$

As firm 1 raises p_1 along H, its expected profit is increasing because $F_2(p_1)$ is constant along H (and equal to $F_2(r_1)$ if $r_1 \in H$). If $r_2 \notin H$, then firm 1 could increase profits by shifting all mass in H slightly below p^u (or to p^u if firm 2 does not have an atom there), a contradiction. If $r_2 \in H$, then firm 1 can increase profits by shifting all mass in (p^d, r_2) to slightly below r_2, and all mass in (r_2, p^u) either to slightly below r_2 or to p^u, again contradicting the equilibrium. A similar argument applies when firm 1 has no support over H, but firm 2 does. This tells us that any breaks in $S_1 \cap S_2$ are mutual.

Now suppose that neither firm randomizes over H in equilibrium. Suppose first that $p^d \neq r_1$, $p^d \neq r_2$ and that neither firm has an atom at p^d. Then either firm 1 has a strictly higher expected profit at p^u (or slightly below r_2 if $r_2 \in H$) than at p^d, or firm 2 has a strictly higher expected profit at p^u (or slightly below r_1 if $r_1 \in H$) than at p^d, or both, if neither firm has an atom at p^u, contradicting the equilibrium.

Suppose that firm i has an atom at $p^d \neq r_j$. Because there are no mutual atoms, firm i could increase profits by shifting its atom to p^u (or slightly below p^u if firm j has an atom there, or slightly below r_j if $r_j \in H$).

If $p^d = r_j \neq r_i$ and firm i has no atom at p^d, firm j's expected profit will be strictly higher at p^u (or slightly below p^u if firm i has an atom there, or slightly below r_i if $r_i \in H$) than at p^d. But if firm i does have an atom at p^d, then it is possible that profits are the same at p^d and p^u for each firm. If $\gamma \neq 1$, firm i can profitably deviate by shifting its atom slightly below p^d. In doing so, it retains $1-\gamma$ non-shoppers who search after observing a price r_j and have a positive probability of purchasing from firm j. However, if $\gamma = 1$, neither firm might have a profitable deviation. This may also be the case if, $p^d = r_2 = r_1$.

By Claim 1, we know that both S_1 and S_2 have the same lower bound $\underline{p}$, so $S_1 \Delta S_2 \in (\min\{\bar{p}_1, \bar{p}_2\}, \max\{\bar{p}_1, \bar{p}_2\}]$. Suppose, without loss of generality, that $\bar{p}_1 > \bar{p}_2$. At $p_1 \in (\bar{p}_2, \bar{p}_1]$, firm 1's expected profit is $p_1\lambda(1-\mu)(\mathbb{I}_{p_1<r_2} + \gamma\mathbb{I}_{p_1=r_2})$. If $\bar{p}_1 > r_2$, then firm 1 can increase profits by shifting mass in $(r_2, \bar{p}_1]$ to r_2 or slightly below it if $\gamma = 0$. If $r_2 \geq \bar{p}_1$, then profits are strictly increasing in $p_1 \in (\bar{p}_2, \bar{p}_1)$, so firm 1 could increase profits by shifting mass in $(\bar{p}_2, \bar{p}_1]$ to $\min\{r_2, v\} - \varepsilon$ for $\varepsilon > 0$ sufficiently small. As a result, $S_1 \Delta S_2 = \{\min\{v, r_2\}\}$. If firm 2 has no atom at $\bar{p}_2$, firm 1's expected profit at $\bar{p}_1$ is $\bar{p}_1\lambda(1-\mu)$, strictly higher than its expected profit of $\bar{p}_2\lambda(1-\mu)$ at $\bar{p}_2$, a contradiction. If firm 2 has an atom at $\bar{p}_2 \neq r_1$, because there are no mutual atoms, firm 2 can profitably shift the atom to slightly below $\bar{p}_1$ (or slightly below r_1 if $r_1 \in (\bar{p}_2, \bar{p}_1]$). However, if $\gamma = 1$, firm 2 has an atom at $\bar{p}_2 = r_1$ and $F_1(r_1)$ is large enough, then neither firm might have a profitable deviation. A similar argument applies when $\bar{p}_2 > \bar{p}_1$. □

Claim 4. *Firm i does not have an atom in the lower bound or the interior of firm j's equilibrium support, except possibly at r_j.*

Proof. Suppose without loss of generality that firm 2 has an atom at $p \in S_1 \backslash \{\bar{p}_1\}$, and suppose that $p \neq r_1$. Firm 1's expected profit at $p - \varepsilon$ when firm 2 charges p is given by Expression (7), whereas its expected profit at $p + \varepsilon$ is

$$(p+\varepsilon)\lambda(1-\mu)(\mathbb{I}_{p+\varepsilon<r_2} + \gamma \mathbb{I}_{p+\varepsilon=r_2}) \tag{9}$$

Expression (9) is smaller than Expression (7) for ε sufficiently small.

Firm 1 can increase expected profits by shifting mass from $(p_2 - \varepsilon, p_2 + \varepsilon]$ to $p_2 - \varepsilon$ for the following reason. For any price that firm 2 charges, shifting mass to $p - \varepsilon$ never reduces the number of sales for firm 1, so it loses at most 2ε. However, when p is charged with positive probability, lowering the price by 2ε or less will, with positive probability, lead to a gain, and with complementary probability, at worst, lead to a loss of 2ε. Therefore, by shifting its mass between p and $p + \varepsilon$ to $p - \varepsilon$ for sufficiently small ε, firm 1 increases its expected profit, a contradiction. □

Claim 5. *If $\bar{p}_1 = \bar{p}_2 = \bar{p}$ then either*

(i) $\bar{p} = \min\{v, r_1, r_2\}$, the supports have no breaks, and at most one firm can have an atom at $\bar{p}$, or

(ii) $\bar{p} = \min\{v, \max\{r_1, r_2\}\}$*, there is a mutual break above* $\min\{r_1, r_2\} < \bar{p}$*, firm* i *has an atom at* r_j*, and firm* j *has an atom at* $\bar{p}$*.*

Proof. Suppose that $\bar{p}_1 = \bar{p}_2 = \bar{p}$ and neither firm has an atom at $\bar{p}$. From Claims 1 and 4 we know that $\underline{p} < \bar{p} \leq v$. Suppose $\bar{p} < \min\{v, r_2\}$. At $\bar{p}$, firm 1 expects profit of $\bar{p}\lambda(1-\mu)$. For $\lambda \neq 0$, by raising its price to $\min\{v, r_2\} - \varepsilon$, firm 1 expects to gain $[\min\{v, r_2\} - \varepsilon - \bar{p}]\, \lambda(1-\mu) > 0$ for sufficiently small $\varepsilon > 0$, a contradiction. Suppose instead $\bar{p} > \min\{v, r_2\}$. But then at $\bar{p}$ firm 1 expects no profit, a contradiction, so $\bar{p} = \min\{v, r_2\}$. If $\lambda = 0$, firm 1 expects no profit at $\bar{p}$ unless either $\bar{p}_1 < \bar{p}_2$ or firm 2 has an atom at $\bar{p}$. By a similar argument, $\bar{p} = \min\{v, r_1\}$, so $\bar{p} = \min\{v, r_1, r_2\}$.

From Claim 2, we know that at most one firm can have an atom at $\bar{p}$, say firm j. If $\gamma = 1$ or $v < r_i$, then following the argument in the previous paragraph, $\bar{p} = \min\{v, r_i\}$. Otherwise, firm j cannot have an atom at $\bar{p}$ (using similar reasoning to that in the proof of Claim 3). Moreover, if $r_j \geq r_i$, then $\bar{p} = \min\{v, r_1, r_2\}$ and from Claim 3, we know that the firm supports have no breaks. Conversely, suppose $r_j < r_i$ (and therefore $r_j < v$). Without loss of generality, let $i = 1$. From Claim 4 we know that firm 2 cannot have an atom at r_2. At r_2, firm 1 expects profit of

$$r_2\{[\mu + \lambda(1-\mu)(1-\gamma)][1 - F_2(r_2)] + \lambda(1-\mu)\gamma\} \tag{10}$$

At $p_1 \in (r_2, \bar{p})$, firm 1 expects profit of

$$p_1[\mu + \lambda(1-\mu)][1 - F_2(p_1)] \tag{11}$$

But because $p_1 \in (r_2, \bar{p})$, by definition, $0 < F_2(r_2) \leq F_2(p_1)$, so for a small enough p_1, Expression (10) will be strictly greater than Expression (11) as long as $\gamma > 0$. Therefore, r_2 must be the lower bound for a break in S_1, so we must be in case (iii) of Claim 3. The second to last paragraph in the proof of Claim 3 implies that this equilibrium only exists for $\gamma = 1$. □

Claim 5 allows us to narrow down the possible supports to item (i) in Claim 3 and the two items in Claim 5. It immediately follows that existence of an atom requires non-shoppers who observe the reservation price in equilibrium to stop searching. Below, we restrict analysis to equilibria where $v \leq r_i$ implies that $v \leq r_j$, $j \neq i$. That is, in the remainder of our analysis, we do not examine the potential existence of equilibria where v is sufficiently large not to influence the search decisions of consumers local to one firm, yet small enough to constrain the decisions of consumers local to the other. Therefore, for the remainder of this proof, suppose that v does not bind in equilibrium. The next two claims rule out item (i) in Claim 3 and item (ii) in Claim 5. The analysis proceeds similarly in the case that v does bind and is left to the reader.

Claim 6. *In equilibrium,* $\bar{p}_1 = \bar{p}_2$.

Proof. From Claims 3 and 4, we know that an equilibrium where $\bar{p}_1 \neq \bar{p}_2$ is characterized by $r_j = \bar{p}_i < \bar{p}_j = r_i = S_j \Delta S_i$. Additionally, each firm i has an atom at $\bar{p}_i$. We show that an equilibrium characterized as such, does not exist, thereby ruling out item (i) in Claim 3.

Without loss of generality, suppose $\lambda \in (1/2, 1]$. Then, $F_1(p)$ and $F_2(p)$ are represented by Equation (2) over $\left[\underline{p}, \bar{p}_1 = r_2^*\right)$, where r_i^* represents the equilibrium reservation price for non-shoppers local to firm $j \neq i$. Note that, $\lambda > 1/2 \Leftrightarrow F_1(p) < F_2(p)$ on $\left(\underline{p}, r_2^*\right)$, so in equilibrium, it must be that $r_2^* = \bar{p}_1 < \bar{p}_2 = r_1^*$. A complete solution to this equilibrium requires the following set of equations to hold.

$$\begin{aligned} &\mathbb{E}\Pi_1(\underline{p}) = \mathbb{E}\Pi_1(p_1, F_2(p_1)) \\ &\Leftrightarrow [\mu + \lambda(1-\mu)]\underline{p} = \{\mu[1 - F_2(p_1)] + \lambda(1-\mu)\}p_1 \end{aligned} \tag{12}$$

$$\begin{aligned}&\mathbb{E}\Pi_1(\underline{p}) = \mathbb{E}\Pi_1(r_2^*, F_2(r_2^*)) \\ &\Leftrightarrow [\mu + \lambda(1-\mu)]\underline{p} = [\mu \Pr(p_2 = r_1^*) + \lambda(1-\mu)]r_2^*\end{aligned} \tag{13}$$

$$\begin{aligned}&\mathbb{E}\Pi_2(\underline{p}) = \mathbb{E}\Pi_2(p_2, F_1(p_2)) \\ &\Leftrightarrow [1 - \lambda(1-\mu)]\underline{p} = \{\mu[1 - F_1(p_2)] + (1-\lambda)(1-\mu)\}p_2\end{aligned} \tag{14}$$

$$\begin{aligned}&\mathbb{E}\Pi_2(\underline{p}) = \mathbb{E}\Pi_2(r_1^*) \\ &\Leftrightarrow [1 - \lambda(1-\mu)]\underline{p} = (1-\lambda)(1-\mu)r_1^*\end{aligned} \tag{15}$$

$$\int_{\underline{p}}^{r_2^*} F_1(p)\mathrm{d}p + (r_1^* - r_2^*) = c \tag{16}$$

$$\int_{\underline{p}}^{r_2^*} F_2(p)\mathrm{d}p = c \tag{17}$$

$$\begin{aligned}&\mathbb{E}\Pi_1(r_2^*, F_2(r_2^*)) > \mathbb{E}\Pi_1(r_1^* - \varepsilon, F_2(r_1^* - \varepsilon))\ \forall \varepsilon \in (0, r_1^* - r_2^*) \\ &\Leftrightarrow [\mu \Pr(p_2 = r_1^*) + \lambda(1-\mu)]r_2^* \geq r_1^* \Pr(p_2 = r_1^*)[\mu + \lambda(1-\mu)]\end{aligned} \tag{18}$$

$$\Pr(p_1 = r_2^*) = 1 - \lim_{\varepsilon \to 0^-} F_1(r_2^* - \varepsilon) \in (0,1) \tag{19}$$

$$\Pr(p_2 = r_1^*) = 1 - F_2(r_2^*) \in (0,1) \tag{20}$$

We use the following procedure to attempt to find an equilibrium. First, we use Equation (12) and Equation (14) to solve for $F_2(p)$ and $F_1(p)$ respectively, in terms of $\underline{p}$. Plugging $F_2(p)$ into Equation (17) and using Equation (13) to solve for $\underline{p}$ we obtain r_2^* in terms of $\Pr(p_2 = r_1^*)$. Plugging $F_1(p)$ into Equation (19) yields $\Pr(p_1 = r_2^*)$ in terms of $\Pr(p_2 = r_1^*)$. Rewriting $F_1(p)$ in terms of $\Pr(p_2 = r_1^*)$ and plugging into Equation (16) yields r_1^* in terms of $\Pr(p_2 = r_1^*)$. Finally, using Equation (15) to solve for $\underline{p}$ and setting this equal to the solution obtained from Equation (13) we can rewrite r_1^* as an alternate function of $\Pr(p_2 = r_1^*)$. Setting the two expressions for

r_1^* equal to each other, we can now solve for $\Pr(p_2 = r_1^*)$ in terms of the exogenous parameters. We can then use this to see if Inequality (18) holds. Using the procedure described, the solution to $\Pr(p_2 = r_1^*)$ obtains implicitly from the following equation:

$$\begin{aligned} & c + r_2^* \Bigg\{ 1 - \frac{[1-\lambda(1-\mu)]\,[1-\Pr(p_2 = r_1^*)]}{\mu + \lambda(1-\mu)} \\ & + \frac{[1-\lambda(1-\mu)]\,[\lambda(1-\mu) + \mu\Pr(p_2 = r_1^*)]}{\mu\,[\mu+\lambda(1-\mu)]} \ln\left[\frac{\mu+\lambda(1-\mu)}{\lambda(1-\mu)+\mu\Pr(p_2 = r_1^*)}\right] \Bigg\} \\ & - \frac{r_2^*\,[\lambda(1-\mu)+\mu\Pr(p_2 = r_1^*)]\,[1-\lambda(1-\mu)]}{[\mu+\lambda(1-\mu)]\,(1-\lambda)(1-\mu)} = 0 \end{aligned} \tag{21}$$

where r_2^* is a function of $\Pr(p_2 = r_1^*)$ and the underlying parameter values. Numerically, for every permissible value of μ and λ and some $\Pr(p_2 = r_1^*) \in (0,1)$, it is possible to check if Equation (21) holds. Doing so, we find that there is no value of $\Pr(p_2 = r_1^*) \in (0,1)$ that would lead Equation (21) to hold, a contradiction. $\square$

In Figure 1 we plot the left hand side of Equation (21) over all permissible values of μ and λ assuming that $\Pr(p_2 = r_1^*) = 1/2$. Observe that the expression is always strictly below zero. A simple Mathematica file that allows readers to plot the same expression for arbitrary values of $\Pr(p_2 = r_1^*)$ is available upon request.

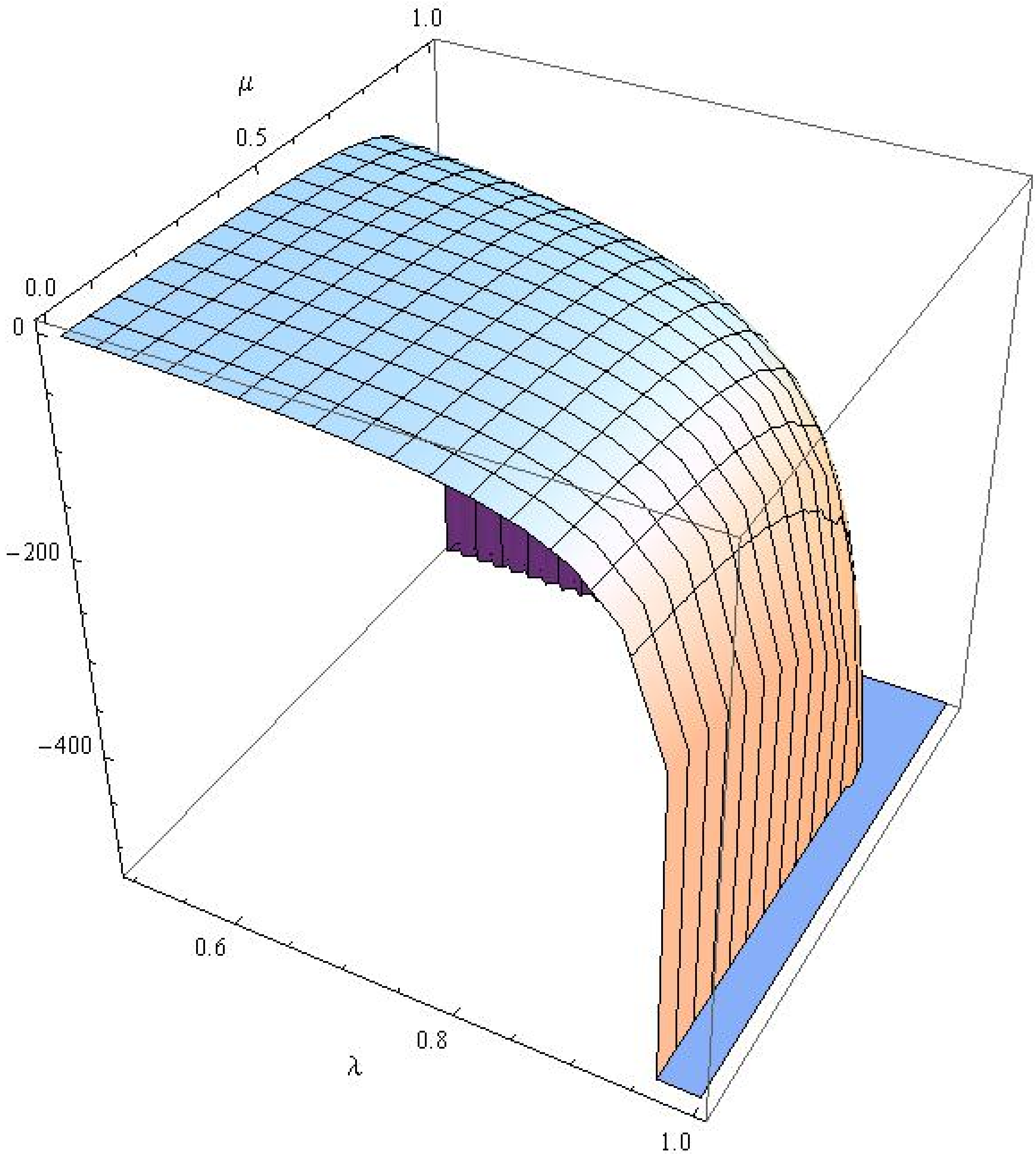
Claim 7. *There are no breaks in the equilibrium supports.*

Proof. From Claims 3 through 6, we know that an equilibrium with breaks is characterized by $r_i < r_j = \bar{p}$ and a mutual break over (r_i, p^u) for $p^u \in (r_i, \bar{p})$. Additionally, each firm i has an atom at r_j, $j \neq i$. Once again assuming that $\lambda \in (1/2, 1]$, and solving for equilibrium following a similar approach to the one used to prove Claim 6, we can obtain an implicit solution to $F_1(r_2^*)$ by setting two expressions for r_1^* equal to each other to find that no solution to $F_1(r_2^*) \in (0,1)$ exists, a contradiction. For completeness, we note that the implicit solution for $F_1(r_2^*)$ can be represented by the following equation:

$$
\begin{aligned}
&p^u \left\{1 - \frac{1-[\mu+\lambda(1-\mu)]F_1(r_2^*)}{(1-\lambda)(1-\mu)}\right\} - (p^u - r_2^*)F_1(r_2^*)\left[\mu+\lambda(1-\mu)\right] \\
&[\mu+\lambda(1-\mu)]\left\{c - \frac{1-\lambda(1-\mu)F_1(r_2^*)}{\mu}\left\{r_2^* - \underline{p}\left[1+\ln\left(r_2^*-\underline{p}\right)\right]\right\}\right. \qquad (22) \\
&\left.-p^u\left[F_1(r_2^*) - \frac{1}{\mu+\lambda(1-\mu)}\right]\ln\left\{\frac{1-[\mu+\lambda(1-\mu)]F_1(r_2^*)}{(1-\lambda)(1-\mu)}\right\}\right\} = 0
\end{aligned}
$$

where r_2^* and p^u are both functions of $F_1(r_2^*)$ and the underlying parameters. A Mathematica file that allows readers to plot the left hand side of Equation (22) for arbitrary values of $F_1(r_2^*)$ is available upon request. $\square$

Figure 1: Left hand side of Equation (21) for $\Pr(p_2 = r_1^*) = 1/2$ over all $\mu \in (0, 1)$ and $\lambda \in (1/2, 1]$.



References

[1] Arbatskaya, M. [2007]. "Ordered Search." *The RAND Journal of Economics* 38, 119-126.

[2] Armstrong, M., Vickers, J., Zhou, J. [2009]. "Prominence and Consumer Search." *RAND Journal of Economics* 40, 209-233.

[3] Benabou, R. [1993]. "Search Market Equilibrium, Bilateral Heterogeneity, and Repeat Purchases." *Journal of Economic Theory* 60, 140-158.

[4] Deneckere, R., Kovenock, D., Lee, R. [1992]. "A Model of Price Leadership Based on Consumer Loyalty." *The Journal of Industrial Economics* 40, 147-156.

[5] Diamond, P.A., [1971]. "A Model of Price Adjustment." *Journal of Economic Theory* 3, 156-168.

[6] Janssen, M.C.W., Moraga-Gonzalez, J.L, Wildenbeest, M.R. [2005]. "Truly Costly Sequential Search and Oligopolistic Pricing." *International Journal of Industrial Organization* 23, 451-466.

[7] Jing, B., Wen, Z. [2008]. "Finitely Loyal Consumers, Switchers and Equilibrium Price Promotion." *Journal of Economics & Management Strategy* 17, 683-707.

[8] Narasimhan, C. [1988]. "Competitive Promotional Strategies." *Journal of Business* 61, 427-449.

[9] Stahl, D.O. [1989]. "Oligopolistic Pricing with Sequential Consumer Search." *American Economic Review* 79, 700-712.

[10] Stahl, D.O. [1996]. "Oligopolistic Pricing with Heterogeneous Consumer Search." *International Journal of Industrial Organization* 14, 243-268.

[11] Varian, H.R. [1980]. "A Model of Sales." *American Economic Review* 70, 651-659.

[12] Weitzman, M.L. [1979]. "Optimal Search for the Best Alternative." *Econometrica* 47, 641-654.

[13] Wolinsky, A. [1986]. "True Monopolistic Competition as a Result of Imperfect Information." *The Quarterly Journal of Economics* 101, 493-511.

[14] Zhou, J. [2010]. "Ordered Search in Differentiated Markets." *International Journal of Industrial Organization* 29, 253-262.